Proposal for a post-doctoral fellowship IMI 2023-2024

Elastic waves in metamaterials with time-modulated interfaces

keywords: waves, continuum mechanics, homogenisation, Bloch-Floquet, scientific computing

1 Research project

1.1 Context

By structuring materials, it is possible to control waves and obtain surprising effects: high-resolution lenses, directional antennas, invisibility cloaks, etc. This is the paradigm of metamaterials, a field of research that has been very active since the early 2000s and which promises many technological developments, for example improving Wi-Fi coverage or insulating buildings against seismic waves.

However, the properties accessible with these materials also have limits, given by the usual laws of physics. Reciprocity is one of these laws [4]. In the linear regime and without flow, a wave transmitted between two points remains the same when the source and receiver are interchanged. Breaking the reciprocity makes it possible to control the waves in a novel way by making directional band gaps possible. Applications include wave filtering, diode design and logic gates in elastodynamics.

1.2 State of the art

Breaking the reciprocity requires special conditions. One possibility is based on time-varying physical properties (figure 1). Long considered a curiosity, the idea of modulating the parameters of continuous media in time is now experimentally feasible, notably with piezoelectric controllers or magneto-rheological materials. However, at low frequencies, non-reciprocity seems to be achieved only if all the physical parameters are modulated in space and time [3], which is restrictive in practice.

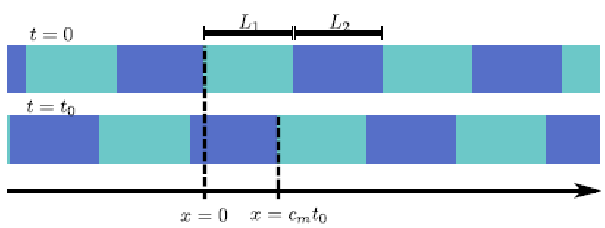

Figure 1: 1D bilaminate medium in the form of periodic travelling waves of velocity c_m, at two different times. Issued from [3].

Rather than modulating the properties of a continuous medium in time, a simpler alternative is to modulate discrete elements coupled to a propagation medium. The aim of this post-doctoral fellowship is to study, using analytical and numerical time methods, the nature of these media in order to evaluate their potential for wave control.

1.3 Work plan

To do this, we propose to focus on imperfect interface laws, which are written in 1D:

$$[\![u]\!]_{x_i} = \frac{1}{K_i} \left\langle\!\left\langle \sigma \right\rangle\!\right\rangle_{x_i}, \qquad [\![\sigma]\!]_{x_i} = M_i \, \partial_{tt}^2 \left\langle\!\left\langle u \right\rangle\!\right\rangle_{x_i}, \tag{1}$$

where u is the displacement, σ the stress, and with stiffnesses $K_i > 0$ and masses $M_i \geq 0$. In (1), $[\![.]\!]_{x_i}$ and $\langle\langle . \rangle\rangle_{x_i}$ are respectively the jump and the mean value at the x_i interface. The coupling of elastodynamics and interface conditions (1) has been studied in several works: high-frequency homogenisation [1], low-frequency periodic homogenisation [2]. Compared to existing work, the novelty will consist in modulating the interface parameters in time: $K_i(g(x_i,t))$, $M_i(h(x_i,t))$. Different families of modulation will be considered, for example $g(x_i,t) = g(x_i - Vt)$ where V is a propagation speed, and g is a periodic function. The work plan is as follows:

- theoretical study of the evolution equations (energy balance, parametric instabilities depending on V);
- periodic homogenisation to order 2, by double-scale asymptotic development;
- Bloch-Floquet analysis of the asymmetry of the bands and gaps as a function of the modulation families;
- high-frequency homogenisation. This will lead to an analytical study of the topological modes;
- study of slowly varying interface parameters, leading to the *rainbow trapping* [5] in time ;
- optimisation of the modulations to obtain a given functionality.

All these theoretical analyses will be illustrated and tested by comparison with numerical simulations. Once the 1D case is completed, the work will be generalised to 2D.

2 Organisation

This subject is aimed at candidates with a thesis in theoretical mechanics, theoretical physics or applied mathematics. Skills in scientific computing are required. The post-doctorate will be carried out at the Laboratoire de Mécanique et d'Acoustique (LMA, Marseille), where it will be supervised by Bruno Lombard. This subject is part of a collaboration between researchers from different laboratories, who will participate in the supervision of the work (applications should be sent to these 4 supervisors):

- Bruno Lombard, Laboratoire de Mécanique et d'Acoustique (France) : `lombard@lma.cnrs-mrs.fr` http://www.lma.cnrs-mrs.fr/spip.php?auteur30&lang=fr
- Raphael Assier, University of Manchester (United Kingdom) : `raphael.assier@manchester.ac.uk` https://raphaelassier.weebly.com/
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- Marie Touboul, Imperial College London (United Kingdom) : `m.touboul@imperial.ac.uk`, https://sites.google.com/view/marietouboul/home

Two 15-day stays will take place, in London and Manchester respectively. Teaching activities will be offered: seminars, doctoral courses, internship supervision.

References

[1] R. Assier, M. Touboul, B. Lombard, C. Bellis, *High-frequency homogenisation in periodic media with imperfect interfaces*, Proc. R. Soc. London A 476 (2020), 20200402.

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[3] H. Nassar, X.C. Xu, A.N. Norris, G.L. Huang, *Modulated phononic crystals: Non-reciprocal wave propagation and Willis materials*, J. Mech. Phys. Sol. 101 (2017), 10-29.

[4] C. Rasmussen, L. Quan, A. Alù, *Acoustic nonreciprocity*, Journal of Applied Physics 129 (2021), 210903.

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